INDIAN STATISTICAL INSTITUTE

Probability Theory I: B. Math (Hons.) I

Semester I, Academic Year 2017-18

Mid-term Exam

Date: 12/09/2017 **Total Marks: 30** **Duration: 10 am - 12 pm**

- **Show all your works and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. Roads A and B are the only escape routes from a state prison. Prison records show that, of the prisoners who tried to escape, 40% used road A, and 60% used road B. These records also show that 80% of those who tried to escape via A, and 70% of those who tried to escape via B were captured.
   (a) (3 marks) What is the chance that one prisoner can successfully escape from the prison?
   (b) (7 marks) Suppose that two prisoners have independently and successfully escaped from the prison. What is the probability that they have used the same road to escape?

2. (8 marks) Let N be the number of empty poles when r flags of different colours are displayed randomly on n poles arranged in a row (here $r, n \in \mathbb{N}$). Assuming that there is no limitation on the number of flags on each pole, compute the expectation of N.

3. (12 marks) Suppose $X \sim Bin(n, p)$. Compute the 3^{rd} moment of X.